Applications of Tensor Decomposition

UMBC REU Site: Interdisciplinary Program in High Performance Computing

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Motivation

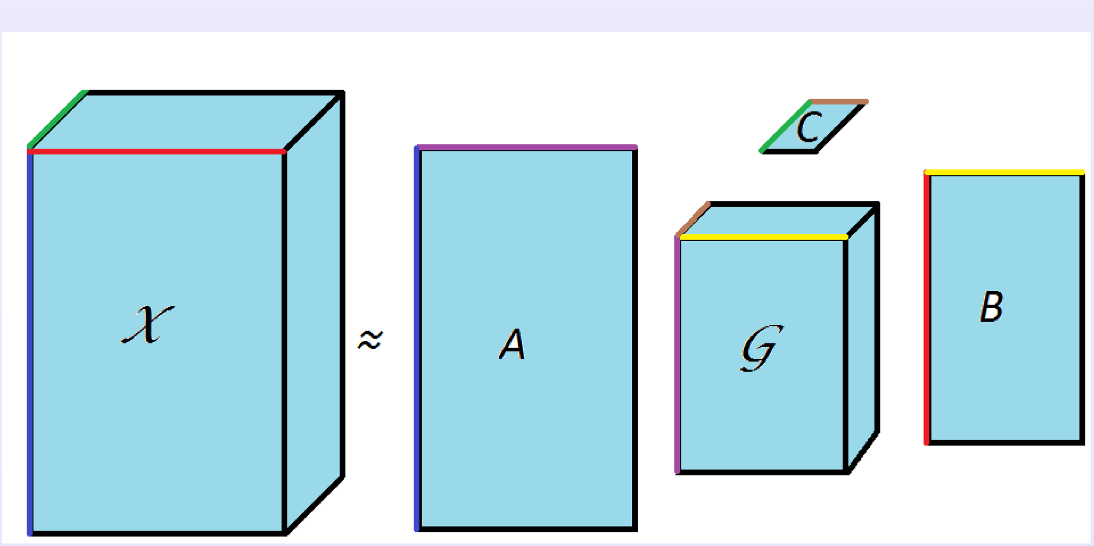
Many application problems in data analysis inherently contain **multidimensional data**, also known as **tensors**. Oftentimes, summaries about the data are desired for a study, for which methods such as principal component analysis are useful. For N-dimensional tensors, an alternative approach is to compute and interpret tensor decompositions of the original multidimensional data.

Tensor Basics

A tensor is an N-way array used to store data, and is thus a generalization of a matrix. A **Tucker decomposition** for a tensor expresses it in terms of components. It is also known as 3MPCA [2], for a 3-way tensor $\mathcal{X} \in \mathbb{R}^{I\times J\times K}$ and it satisfies

$$\mathcal{X} \approx \mathcal{G} \times_1 A \times_2 B \times_3 C$$

for the core tensor $\mathcal{G} \in \mathbb{R}^{P\times Q\times R}$ and the component (orthogonal) matrices $A \in \mathbb{R}^{I\times P}$, $B \in \mathbb{R}^{J\times Q}$, $C \in \mathbb{R}^{K\times R}$, with user requested positive integers $P \le I$, $Q \le J$, $R \le K$.



Matlab Tensor Toolbox

Starting from an initial guess, the `tucker_als` function from the Matlab Tensor Toolbox [4] iteratively attempts to fit an alternating least squares (ALS) model to compute a Tucker decomposition with a desired core tensor size.

Tucker Decomposition Results

Example: Psychological experiment [3]:

- $I = 326$ children who exhibited
- $J = 5$ behaviors — *Proximity Seeking (PS), Contact Maintaining (CM), Resistance (R), Avoidance (AV), and Distance Interaction (DI)* — in
- $K = 2$ situations.

⇒ Data is 3-way tensor $\mathcal{X} \in \mathbb{R}^{326\times 5\times 2}$.

Strength of each behavior is scored from 1 to 7, resulting in data, for instance, for the first five children in situation 1:

Child	PS	CM	R	AV	DI
1	3	2	1	2	7
2	6	7	1	1	1
3	1	2	1	2	7
4	7	7	7	1	1
5	6	4	4	1	1

Computing a Tucker decomposition using `tucker_als` with requested core tensor size $2\times 2\times 2$ gives the core tensor $\mathcal{G} \in \mathbb{R}^{2\times 2\times 2}$ and component matrices $A \in \mathbb{R}^{326\times 2}$, $B \in \mathbb{R}^{5\times 2}$, $C \in \mathbb{R}^{2\times 2}$

$$\mathcal{G}(:,:,1) = \begin{bmatrix} 0.3376 & 16.3568 \\ -1.7602 & 0.7665 \end{bmatrix}$$

$$\mathcal{G}(:,:,2) = \begin{bmatrix} 178.6889 & -0.6870 \\ -0.0978 & -80.4706 \end{bmatrix}$$

$$A = \begin{bmatrix} 0.0476 & -0.0663 \\ 0.0571 & 0.0811 \\ \vdots & \vdots \\ 0.0556 & -0.0236 \\ 0.0547 & -0.0455 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.5444 & -0.3705 \\ 0.4363 & -0.5090 \\ 0.3391 & -0.1313 \\ 0.3919 & 0.3124 \\ 0.4947 & 0.6992 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.7342 & 0.6789 \\ -0.6789 & 0.7342 \end{bmatrix}$$

Each component matrix allows us to separate properties. For instance, the second column of B noticeably groups the first three behaviors of PS, CM, and R, and the last two behaviors of AV and DI (by looking at sign and magnitude).

Interpreting Tucker Results

The column vectors in B are principal components. The projection of the first five children's data onto the second column of B is

$$\mathcal{X}(1:5,:,1)\,B(:,2) = \begin{bmatrix} 3 & 2 & 1 & 2 & 7 \\ 6 & 7 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 & 7 \\ 7 & 7 & 7 & 1 & 1 \\ 6 & 4 & 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} -0.3705 \\ -0.5090 \\ -0.1313 \\ 0.3124 \\ 0.6992 \end{bmatrix} = \begin{bmatrix} 3.2583 \\ -4.0955 \\ 3.9993 \\ -6.0638 \\ -3.7725 \end{bmatrix}$$

The significance of the resulting vector is that negative values correspond to the extent to which behaviors of PS and/or CM are present, whereas positive values correspond to the extent to which behaviors of AV and/or DI are present. Thus, the projections summarize information about the behavior of each child in situation 1.

References

[1] Kolda and Bader, Tensor Decompositions and Applications, *SIAM Review*, 2009

[2] Kiers and Mechelen, Three-Way Component Analysis: Principles and Illustrative Application, *Psychological Methods*, 2001

[3] The Three-Mode Company: `three-mode.leidenuniv.nl` > Data sets > Dutch children in the Strange Situations

[4] Matlab Tensor Toolbox: `www.sandia.gov/~tgkolda/TensorToolbox/`

[5] Full technical report: HPCF–2016–17, `hpcf.umbc.edu` > Publications

Acknowledgments

- REU Site: `hpcreu.umbc.edu`
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